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This experiment demonstrated the obtainable accuracy for antenna position using Doppler techniques and the NAVSTAR The antenna was periodically moved to three locations approx moves occurred within a one minute Doppler count interval least six Doppler measurements were made for each location of used to determine the observed change in range from the antenn	Global Positioning System (GPS). Simately 2 to 3 meters apart. The every fifteenth interval until at change. These measurements were
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the movement of the antenna. The calculated change in range was computed from the surveyed distance moved and the angular differences in elevation and azimuth between the direction of the move and the direction of the satellite. The observed and calculated changes in range were compared. The three-dimensional antenna position changes were estimated using six observed change in range measurements for each location change. The estimated results were compared with surveyed values and demonstrated subdecimeter accuracy.

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# **FOREWORD**

The NAVSTAR Geodetic Receiver System (NGRS) was developed at the Naval Surface Weapons Center using a Global Positioning System (GFS) receiver built by Stanford Telecommunications Incorporated. This report presents the first application of this system to detect and estimate changes in the receiver's antenna position using Doppler techniques.

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#### I. INTRODUCTION

This experiment demonstrated the obtainable accuracy using Doppler techniques for estimating position changes of a Global Positioning System receiver antenna.

Accurate Doppler measurements were taken using the NAVSTAR Geodetic Receiver System (NGRS), developed at the Naval Surface Weapons Center. The system uses a GPS receiver built by Stanford Telecommunications Incorporated of Sunnyvale, California, and a Hewlett-Packard 506/A high performance cesium clock.

The experiment consisted of periodically moving the receiver antenna from its original position (location 0 of Figure 1) to approximately 2 meters (m) east of the original position (location 1) and then to a third position approximately 2 m south of the original position (location 2). Figure 1 presents the surveyed relative locations.

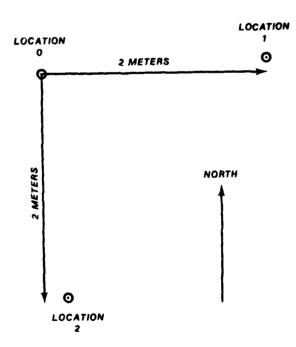


Figure 1. Relative Antenna Locations

Each antenna move was timed to be within a one-minute Doppler count interval. The moves occurred every fifteenth interval, except during a period between tracking of the two satellites used to collect data. The moves were repeated until at least six measurements were made to each location. These measurements were used to estimate the three-dimensional position change of the antenna. This emulated a multichannel receiver tracking six satellites simultaneously.

The next section presents the calculated change in range due to the antenna position change only. Section III gives the procedure for obtaining the observed change in range and compares it with the calculated values. Section IV describes the procedure for estimating the three-dimensional antenna position change, and Section V presents the results. The accuracies of the experiment are discussed in the final section.

### II. CALCULATED CHANGE IN RANGE

The calculated change in range from the satellite to the antenna due to the antenna position change  $\Delta R_c$  is shown geometrically in Figure 2. Parallel lines are assumed from both locations to the satellite since the distance to the satellite is much larger than the distance moved. The figure shows the  $\Delta d$  baseline direction to be  $\alpha$  degrees from north. The values of  $\alpha$  for the three position changes of the experiment are given in Table 1. For example, for location 0 to location 1  $\alpha$  is 90°. The survey values for the distance moved in the  $\alpha$  direction,  $\Delta d$ , and the perpendicular distance,  $\Delta p$ , are also given in the table. The antenna was moved in the horizontal plane so that the change in altitude,  $\Delta h$ , is zero. The accuracy of the survey values should be within 1.0 centimeter (cm).

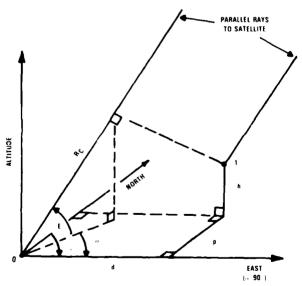


Figure 2. Geometric Representation of Position Change from Location 0 to Location 1

Table 1. Surveyed Change in Antenna Position (m)

Location Change	α (°)	_∆d*	Δ <b>p</b> *	Δh*
0 to 1	90	2.000	0.133	.000.
1 to 2	225	2.779	0.238	.000
2 to 0	0	2.000	0.203	.000

<sup>\*</sup>Accuracy to within 1 cm

The relationship for the change in range is found from a first-order expansion about the origin (location 0).

$$\Delta R_{c}(t_{i}) = R_{c}(t_{i}, d + \Delta d, p + \Delta p, h + \Delta h) - R_{c}(t_{i}, d, p, h)$$
 (1)

$$= \frac{\partial R_c}{\partial d} (t_i, d, p, h) \Delta d + \frac{\partial R_c}{\partial p} (t_i, d, p, h) \Delta p + \frac{\partial R_c}{\partial h} (t_i, d, p, h) \Delta h.$$
 (2)

From Figure 2, the change in calculated range for small position changes from the origin are given as follows

$$\frac{\partial R_{c}(t_{i}, d, p, h)}{\partial d} \bigg|_{d=p \approx h=0} = -\cos \theta(t_{i}) \cos E(t_{i})$$
(3)

$$\frac{\partial R_{c}(t_{i}, d, p, h)}{\partial p} \bigg|_{\mathbf{d} = p = h = 0} = -\sin\theta(t_{i})\cos E(t_{i})$$
(4)

$$\frac{\partial R_{c}(t_{i}, d, p, h)}{\partial h} \bigg|_{d=p\approx h=0} = -\sin E(t_{i})$$
 (5)

where

 $t_i$  = time of the expansion  $E(t_i)$  = elevation to the satellite  $A_z(t_i)$  = azimuth to the satellite  $\alpha$  = direction of the base line  $\theta(t_i) = \alpha - A_z(t_i)$   $\Delta d$  = survey value for change in position in the  $\alpha$  direction  $\Delta p$  = survey value for change in position in the  $\alpha$  - 90° direction  $\Delta h$  = survey value for change in position in the vertical direction

#### III. OBSERVED CHANGE IN RANGE

There are several steps in the procedure to compute the observed change in range,  $\Delta R_0$ . The Doppler measurement is obtained by integrating over a time interval,  $\{t_1, t_2\}$ , the received frequency,  $f_r$ , subtracted from a precise ground station frequency,  $f_g$ . The received frequency is the sum of the transmitted frequency,  $f_T$ , and the Doppler effect. Therefore, the Doppler measurement is

$$\Delta N = \int_{t_1}^{t_2} [f_g - f_r(t)] dt = \int_{t_1}^{t_2} (f_g - f_T + \frac{f_T \dot{R}(t)}{c}) dt$$
 (6)

$$= (f_g - f_T) (t_2 - t_1) + \frac{f_T}{c} [R(t_2) - R(t_1)]$$
 (7)

where c is the speed of light and R(t) is the instantaneous range from the satellite to the station antenna. Note, the Doppler measurement is a difference in ranges at the beginning and end times of the Doppler count interval. The range at any time between these times is immaterial. For this reason, the station antenna for the experiment was in place at its first location at  $t_1$ , moved after  $t_1$ , and put in place at its second location before  $t_2$ . The start and end times for each position change, as well as their corresponding azimuth and elevations, are given in Table 2.

The GPS satellites transmit two signals at frequencies that enable a correction to be made for the first-order ionospheric effect. After ionospheric correction of the range difference—biased observed ranges  $R_0(t_i)$  are formed by adding successive range differences to a nominal range bias. Next, trajectories are computed for the satellites, and calculated ranges  $R_c(t_i, d, p, h)$  are determined to location 0 of Figure 1. Next, a least-squares fit is made using the observed and the calculated ranges to determine the unknown bias  $R_B$  and improvements to six orbit parameters and tropospheric refraction corrections. The residuals of the fit are plotted in Figure 3 for satellite 5 and in Figure 4 for satellite 6.

Table 2. Azimuths and Elevations

Location	Satellite Number	t <sub>i-1</sub> *	Azimuth $A_z(t_{i-1})(^{\circ})$	Elevation E(t <sub>i-1</sub> )(°)	t <sub>i</sub> *	Azimuth $A_z(t_i)(^{\circ})$	Elevation $E(t_i)(^{\circ})$
0-1	5	40500	342.8	54.4	40560	343.6	54.6
-		43200	23.2	60.6	43260	24.2	60.7
		45900	70.2	59.1	45960	71.2	58.9
		48600	107.2	48.2	48660	107.9	47.9
		51300	130.5	31.9	51360	130.9	31.5
	6	60300	61.3	23.0	60360	60.8	23.1
		63000	39.7	19.7	63060	39.3	19.5
1-2	5	41400	355.1	57.0	41460	355.9	57.2
		44100	38.8	61.2	44160	39.9	61.2
		46800	84.2	56.4	46860	85.1	56.1
		49500	116.3	43.2	49560	116.8	42.8
		52200	136.1	25.9	52260	136.5	25.5
	6	61200	76.2	19.5	61260	75.7	19.6
		63900	53.7	23.1	63960	53.2	23.0
2-0	5	42300	8.5	59.1	42360	9.4	59.3
		45000	54.8	60.7	45060	55.8	60,6
		47700	96.6	52.7	47760	97.3	52,4
		50400	124.0	37.7	50460	124.5	37.3
	6	62100	68.9	21.8	62160	68.4	21.9
		64800	46.4	21.9	64860	45.9	21.8

<sup>\*</sup>Year 1980, Day 114 (GMT); unit, second of week

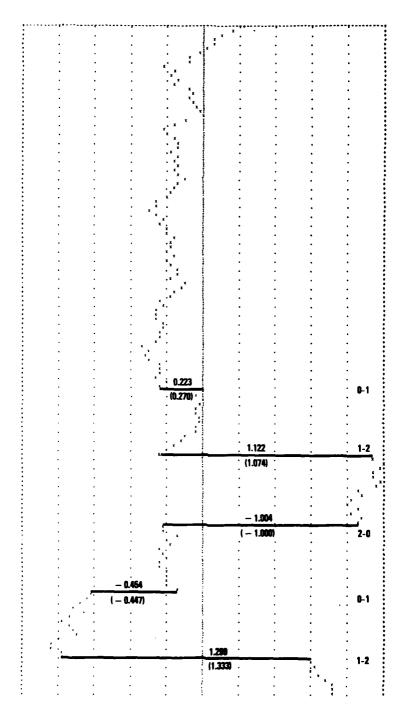


Figure 3. Range Residuals Versus Time for Satellite 5

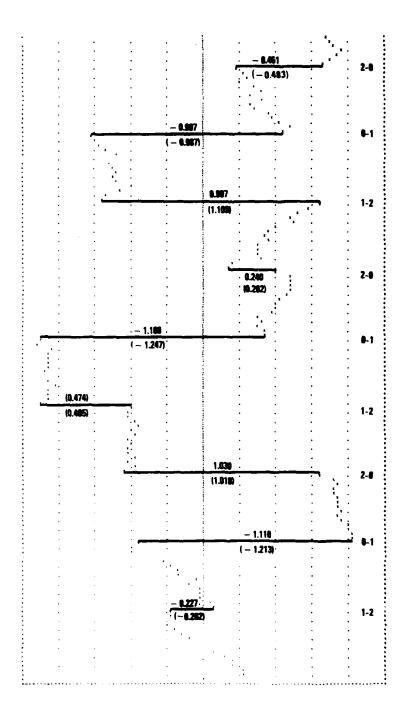


Figure 3. Range Residuals Versus Time for Satellite 5 (Continued)

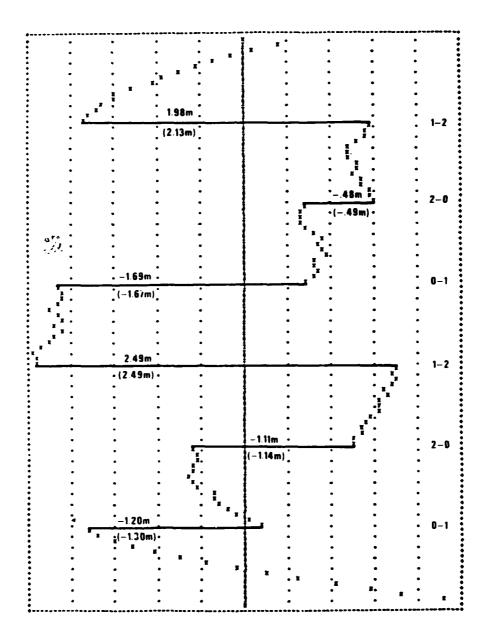


Figure 4. Range Residuals Versus Time for Satellite 6

These plots consist of the least-square range residuals in meters on the abscissa and sequential time on the ordinate, increasing downward. Since the computational fit procedure assumed that the antenna did not move, discontinuties occur at the time intervals when the antenna is moved from each location. The location changes are given in the right-hand column of the figures. These discontinuties are the changes in range due to the antenna position change plus measurement errors. While tracking satellite 5, the antenna was initially placed at location 0 of Figure 1. The first discontinuty of Figure 3 represents the first move of the antenna from location 0 to location 1. The antenna remained at location 1 for fifteen 1-minute Doppler count intervals and was then moved to location 2. These moves were continued clockwise around Figure 1 and are denoted in Figure 3 until the end of the pass. The position changes were then continued for satellite 6 for its entire pass, as shown in Figure 4.

The observed change in range is taken as the difference in the residuals before and after the position change; i.e.

$$\Delta R_{0}(t_{i}, t_{i-1}) = R_{0}(t_{i}) - R_{c}(t_{i}, d, p, h)\Big|_{d=p=h=0} - [R_{0}(t_{i-1}) - R_{c}(t_{i-1}, d, p, h)\Big|_{d=p=h=0}]$$
(8)

where the antenna move occurred after  $t_{i-1}$  and before  $t_i$ . The observed and calculated change in range values are given both in Figures 3 and 4 at the discontinuities and in Table 3. In Figures 3 and 4 the observed values are above the discontinuities and the calculated values are below, in parenthesis. Both values are in units of meters. Also given in Table 3 are the satellite numbers and the start time of the Doppler count interval. The observed and calculated changes in range are differenced for each measurement. The mean and standard deviations of  $(\Delta R_o - \Delta R_c)$  for each location change are also given in Table 3. The combined mean is 0.0016 meters and the combined standard deviation is 0.0573 meters.

Note that the standard deviation of the location change 2-0 was significantly smaller than the two other location changes. By observing Figures 3 and 4, note that the location changes 2-0 always occured at least one-half hour from either end of the satellite pass. The processing algorithm, from which these residuals were obtained, attempts to produce a zero mean result. Consequently, there is a steep slope to the residuals at the beginning and end of the passes. This phenomenon is very severe for the short pass of satellite 6, less so for satellite 5, but casts some doubt on the accuracy of the first and last observations in both passes.

The individual observed minus calculated range differences are divided by the cosine of the corresponding elevation angle and plotted along the azimith to the satellite from location 1 (see Figure 5). These represent a zero elevation plot of the range difference errors. If the survey and measurements were perfect, all the difference values would be at the antenna location. This plot depicts the orientation of the satellites with respect to the base line between location 0 and 1.

Table 3. Observed and Calculated Change in Range

ocation	Satellite Number	_t <sub>i-1</sub> *	$\frac{\Delta R_0(t_i, t_{i-1})(m)}{2}$	$\Delta R_c(m)$	$\Delta R_0 - \Delta R_c(m)$	
0-1	6	60300	-1.692	-1.674	018	
		63000	-1.207	-1.299	0.092	
	5	40500	0.223	0.270	047	$\overline{\Delta R_0 - \Delta R_c} = 0.028$
		43200	454	447	007	0 - c
		45900	997	989	008	$\sigma_{\Delta R_0 - \Delta R_c} = 0.062$
		48600	-1.168	-1.247	0.079	ΔR <sub>0</sub> -ΔR <sub>c</sub>
		51300	-1.110	-1.218	0.108	
1-2	6	61200	1.980	2.125	145	
		63900	2,490	2,494	~.004	
	5	41400	1.222	1.074	0.048	$\overline{\Delta R_0 - \Delta R_c} =033$
		44100	1.299	1.333	044	U c
		46800	0.997	1.109	112	$\sigma_{\Delta R_c - \Delta R_c} = .072$
		49500	0.474	0.485	0.011	ΔR <sub>c</sub> -ΔR <sub>c</sub>
		52200	227	262	0.035	
2-0	6	62100	480	493	0.013	
		64800	-1.110	-1.143	0,033	$\overline{\Delta R_0 - \Delta R_c} = .011$
	5	42300	-1.004	-1.000	004	o c
		45000	451	483	0.032	$\sigma_{\Delta R_0 - \Delta R_c} = .021$
		47700	.240	0.262	022	O AR C
		50400	1.030	1,018	0.012	

<sup>\*</sup>Year 1980, Day 114 (GMT); unit second of week.

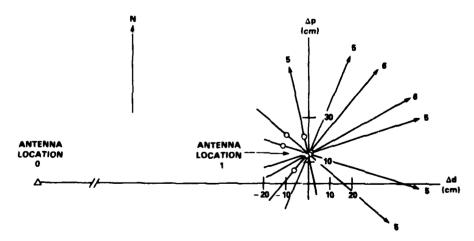


Figure 5. Change in Range Residuals Along Azimuth to the Satellite with Elevation Effect Removed

#### IV. ESTIMATION OF CHANGE IN POSITION

This section describes the procedures for estimating the change in antenna position using the observed change in ranges, described in the previous section, as data. The relationship for  $\Delta R_0(t_i, t_{i-1})$  of (8) is used to create a linear equation in terms of the unknown position changes  $\Delta d$ ,  $\Delta p$ , and  $\Delta h$ . This is done by substituting the appropriate relationship of either  $R_c(t_{i-1}, d + \Delta d, p + \Delta p, h + \Delta h)$  or  $R_c(t_i, d + \Delta d, p + \Delta p, h + \Delta h)$ , evaluated at d = p = h = 0, into (8) to match the specific location change. This is explained below for the three location changes.

## A. LOCATIONS 0 TO 1

From (1) and (2), with evaluations at the origin or location 0,

$$R_c(t_i, d, p, h) = R_c(t_i, d + \Delta d, p + \Delta p, h + \Delta h)$$

$$-\frac{\partial R_c}{\partial d}(t_i, d, p, h) - \frac{\partial R_c}{\partial p}(t_i, d, p, h) - \frac{\partial R_c}{\partial h}(t_i, d, p, h)$$
(9)

is substituted into (8). Using the partial derivatives of (3), (4), and (5) this reduces to the following linear equation in terms of position change parameters  $\Delta d_{01}$ ,  $\Delta p_{01}$ , and  $\Delta h_{01}$ , which are to be estimated. Here, the subscripts denote the specific location change.

$$\Delta R_{0}(t_{i}, t_{i-1}) = -\cos\theta (t_{i}) \cos E(t_{i}) \Delta d_{01} - \sin\theta (t_{i}) \cos E(t_{i}) \Delta p_{01}$$
$$-\sin E(t_{i}) \Delta h_{01} + e(t_{i}, t_{i-1})$$
(10)

where the measurement error is

$$e(t_{i}, t_{i-1}) = R_{0}^{(1)}(t_{i}) - R_{0}^{(1)}(t_{i}, d + \Delta d_{01}, p + \Delta p_{01}, h + \Delta h_{01})$$

$$- [R_{0}^{(0)}(t_{i-1}) - R_{0}^{(0)}(t_{i-1}, d, p, h)]$$
(11)

and where the superscripts indicate the location of the evaluation.

## B. LOCATIONS 2 TO 0

For this case the antenna is at location 2 at  $t_{i-1}$ . Therefore, the expansion of (1) and (2) is evaluated at  $t_{i-1}$ . As above for (9),

$$R_c(t_{i-1}, d, p, h) = R_c(t_{i-1}, d + \Delta d, p + \Delta p, h + \Delta h)$$

$$-\frac{\partial R_c}{\partial d}(t_{i-1}, d, p, h) - \frac{\partial R_c}{\partial p}(t_{i-1}, d, p, h) - \frac{\partial R_c}{\partial h}(t_{i-1}, d, p, h)$$
(12)

The following equations are similar to equations (10) and (11) except for parameters  $\Delta d_{20}$ ,  $\Delta p_{20}$  and  $\Delta h_{20}$ .

$$\Delta R_0(t_i, t_{i-1}) = \cos \theta(t_{i-1}) \cos E(t_{i-1}) \Delta d_{20} + \sin \theta(t_{i-1}) \cos E(t_{i-1}) \Delta p_{20}$$

+ 
$$\sin E(t_{i-1}) \Delta h_{20}$$
 +  $e(t_i, t_{i-1})$  (13)

where the measurement error is

$$e(t_{i}, t_{i-1}) = R_{0}^{(0)}(t_{i}) - R_{c}^{(0)}(t_{i}, d, p, h)$$

$$- [R_{0}^{(2)}(t_{i-1}) - R_{c}^{(2)}(t_{i-1}, d + \Delta d_{20}, p + \Delta p_{20}, h + \Delta h_{20})]$$
(14)

and where the superscripts indicate the location of the evaluation.

#### C. LOCATIONS 1 TO 2

For this case both Equations (9) and (12) are substituted into (8) since both locations are away from the origin. This produces linear equations in terms of the position parameters to be estimated  $\Delta d_{12}$ ,  $\Delta p_{12}$  and  $\Delta h_{12}$  and in terms of  $\Delta d_{01}$ ,  $\Delta p_{01}$  and  $\Delta h_{01}$ . Survey values are used for  $\Delta d_{01}$ ,  $\Delta p_{01}$  and  $\Delta h_{01}$  since the antenna positions are assumed to be known perfectly before each move. The resulting relationship is

$$\Delta R_{0}(t_{i}, t_{i-1}) = -\cos\theta (t_{i}) \cos E(t_{i}) \Delta d_{12} - \sin\theta (t_{i}) E(t_{i}) \Delta p_{12}$$

$$-\sin E(t_{i}) \Delta h_{12} + [\cos\theta (t_{i-1}) \cos E(t_{i-1})$$

$$-\cos\theta (t_{i}) \cos E(t_{i})] \Delta d_{01} + [\sin\theta (t_{i-1}) \cos E(t_{i-1})$$

$$-\sin\theta (t_{i}) \cos E(t_{i})] \Delta p_{01} + [\sin E(t_{i-1})$$

$$-\sin E(t_{i})] \Delta h_{01} + e(t_{i}, t_{i-1})$$
(15)

where the measurement error is

$$\begin{split} e(t_{i}, t_{i-1}) &= R_{0}^{(2)}(t_{i}) - R_{c}^{(2)}(t_{i}, d + \Delta d_{01} + \Delta d_{12}, p + \Delta p_{01} + \Delta p_{12}, \\ & h + \Delta h_{01} + \Delta h_{12}) - [R_{0}^{(1)}(t_{i-1}) - R_{c}^{(1)}(t_{i-1}, d + \Delta d_{01}, \\ & p + \Delta p_{01}, h + \Delta h_{01})] \end{split} \tag{16}$$

and again where the superscripts indicate the location of the evaluation.

#### V. RESULTS

The linear equations of (10), (13), and (15) were used to estimate the three-dimensional position changes in terms of  $\Delta d$ ,  $\Delta p$ , and  $\Delta h$ , which are defined in Section 2.

Observed changes in range  $R_0(t_i, t_{i-1})$  of Table 3 and appropriate azimuths and elevations of Table 2 were substituted into these equations to form an over-determined set of linear equations. Six measurements were used for each position change in order to emulate a multichanneled receiver tracking six satellites. (For locations 2 to 0 all six measurements were used; for both locations 0 to 1 and 1 to 2, which had seven measurements, one point near the end of the pass was eliminated.) The least-squares solutions for the three location changes are given in Table 4 together with the corresponding surveyed values of the position changes. The estimation errors, the differences between the estimated and suveyed values, are also given. The magnitudes of these errors are 3.0, 5.4, amd 6.3 cm, for position changes 2 to 0, 1 to 2, and 0 to 1, respectively. This demonstrates the subdecimeter accuracy of the receiver.

The standard deviations on the individual estimates are also presented in Table 4. These were determined (see Reference 1) by weighting the square roots of the diagonal terms of the 3 x 3 inverse of the parameter coefficient matrix by standard deviations of  $(\Delta R_0 - \Delta R_c)$ , given in Table 4 for each location change. The estimates are consistent with their standard deviations except that the altitude errors are smaller, compared to the  $\Delta d$  and  $\Delta p$  errors, than the standard deviations indicate.

The last three columns of Table 4 have the estimation errors for  $\Delta d$  and  $\Delta p$  converted to errors in north and east directions and the altitude error repeated. The estimation error for "completing the loop" is the sum of direction errors for each leg. Here, the antenna location is assumed to be known perfectly at the beginning of each leg. The resulting error is 3 cm north, - 4 cm south and less than - 1 cm in altitude.

Table 4. Change in Position Results\*

LOC	PARAM	EST VALUE (m)	SUR VALUE (m)	EST -SUR (m)	ST DEV. OF EST (m)	ERROR North (m)	ERROR EAST (m)	ERROR ALTITUDE (m)
0 - 1	Δđ	1940	2 000	- 060	.051		~ 060	
	Δρ	151	133	018	.058	.018	<b>j</b>	İ
	Δh	.001	.000	.001	049			.601
1 - 2	Δd	2.727	2 779	052	.074	.037	.037	
	Δр	.244	.238	.006	.048	004	.004	}
	Δh	014	008	- 014	065			014
2 - 0	Δd	1.979	2.000	021	.023	~.021		
	Δр	223	203	028	822	1	~.020	[
	Δh	.000	.000	.008	.020	1		.001
					TOTAL	030	039	005

<sup>\*</sup>Antenna position assumed to be known perfectly at beginning of each move. Six data points for each location (one end point was deleted for position changes 0-1 and 1-2).

#### VI. CONCLUSION

The experiment demonstrated subdecimeter change in range measurement capability and three-dimensional antenna position change estimation accuracy. Note that very few errors, which would exist for an absolute position test or a two-receiver relative position test (see Reference 2), entered into this change in position experiment. Due to the small baseline absolute orbit errors, atmospheric errors and clock bias errors have very little effect on the results. The three main contributing errors of the experiment are jitter in the measurement of number of cycles in the Doppler count, drift in the satellite and receiver clocks over the 1-minute Doppler count interval, and change in orbit error over the Doppler count interval. The standard deviation of the jitter in the measurement and the combined clock drifts were expected to be about 1 and 2.5 cm, respectively. The change in orbit error appeared to be small over the major portion of the trajectory, but due to the processing anomoly discussed in Section III, the residuals may not be as accurate at the pass extremes as in the center. Note that for the 2 to 0 location change, measurement data was not taken at the beginning or end of the pass. Consequently, the accuracy of the 2 to 0 location change results are significantly better for this change in location.

The sampled standard deviations of the change in range measurement errors for locations 2 to 0, 1 to 2 and 1 to 0 were 2.1, 7.2, and 6.3 cm, respectively. These standard deviations are based only on six or seven measurements. The combined mean and standard deviation based on 20 measurements is 0.2 and 5.7 cm, respectively.

The three-dimensional position change estimation error is the difference between the estimated and surveyed values. The magnitudes of the estimation error for position changes 2 to 0, 1 to 2, and 0 to 1 were 3.0, 5.4, and 6.3 cm, respectively.

These are fairly impressive results even though the baseline was very small and few errors entered into the experiment. Further testing will be done for larger baselines and with more than one receiver.

#### VII. REFERENCES

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